

SESSION G:

CIVIL ENGINEERING

Evolution of nonlinear analysis of RC sections: From iterative computations to rapid moment–curvature prediction using ANN models

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ABSTRACT

Nonlinear moment–curvature ($M-\phi$) analysis is essential for evaluating the strength and ductility of reinforced concrete (RC) structures. Traditional numerical methods, such as iterative fiber analysis, provide high accuracy but are computationally intensive. This paper presents the evolution of approaches – from empirical and classical numerical methods to modern artificial intelligence (AI)-based models. Examples of applying artificial neural networks (ANN) and hybrid ANFIS models for predicting the nonlinear response of beams, columns, and beam–column joints are analyzed. Special attention is given to model architecture, input–output parameters, and prediction accuracy. The paper discusses differences between pointwise and parametric models, as well as recent approaches enhancing ANN interpretability. The study provides a critical review of ANN applications in defining moment–curvature relationships and identifies directions for future research in structural engineering.

KEYWORDS

Moment-curvature relationship, Reinforced concrete, Artificial Neural Networks, ANFIS, Pointwise models, parametric models

1. INTRODUCTION

Nonlinear moment–curvature ($M-\phi$) analysis represents a fundamental step in the accurate assessment of the load-bearing capacity, stiffness, and ductility of reinforced concrete (RC) members subjected to bending. Defining the moment–curvature relationship for reinforced concrete sections is a key problem in bending analysis and is the basis for calculating the deformation capacity of individual sections and entire structural elements. The moment–curvature relationship describes the behavior of the cross-section under monotonically increasing bending moment, where the curvature increases depending on the nonlinear material response. This relationship is particularly important for understanding the behavior of structures subjected to seismic loading, where ductility and energy dissipation capacity are crucial design parameters [1].

In the traditional numerical approach, the moment–curvature relationship is obtained using methods such as iterative section analysis, fiber analysis, and nonlinear finite element methods. These methods provide high accuracy but can be computationally intensive [2] and difficult to generalize for different cross-section configurations and material parameter variations [3]. While methods such as the Newton-Raphson procedure or numerical integration over fiber cross-sections (e.g., in OpenSees) represent the gold standard for structural analysis, their application in real-time design or optimization of a large number of cross-sections becomes impractical. Precisely for this reason, in recent decades, there has been a growing interest in civil engineering in applying ANN models as fast surrogate models capable of approximating non-linear relationships between input parameters and cross-sectional responses without repeating the complete iterative procedure for each new case and eliminating the need for complex experimental or numerical analyses, which is also confirmed by contemporary review papers indicating a widespread and increasing application of ANN methods in structural analysis and design [4, 5].

In civil engineering practice, ANNs have already been successfully used for predicting various performance characteristics of concrete members, including cross-sectional capacity, moment–curvature prediction, deformations, strength, and behavior under damage conditions [1]. The current application of ANNs encompasses a wide range of RC elements, from rectangular beams strengthened with polymers [3] to columns subjected to biaxial bending [4] and complex beam-column joints [5]. The diversity of the applied architectures, ranging from standard multilayer feedforward networks with backpropagation to hybrid ANFIS models, indicates a continuous evolution of the methodology aiming to achieve a correlation higher than 95% relative to numerical and experimental data.

Given the wide range of methods and approaches used in the literature – from classical numerical models to modern ANN and neuro-fuzzy systems – there is a need for a review paper that will consolidate existing findings, systematize types of networks and their applications, and identify the advantages and limitations of different approaches. The objective of this paper is to present a systematic overview and comparative analysis of the application of Artificial Neural Networks (ANN) for predicting the nonlinear moment–curvature relationship of reinforced concrete cross-sections, with the aim of identifying key factors (network type, database, input variables) that affect model accuracy and practical applicability in structural engineering.

2. DEVELOPMENT OF COMPUTATION AND DEFINITION OF THE MOMENT–CURVATURE RELATIONSHIP

The moment–curvature relationship represents a fundamental concept in the bending analysis of reinforced concrete sections, as it describes the relationship between the bending moment and the curvature of the section under loading. The theoretical development of numerical determination of the moment–curvature (M – ϕ) relationship represents an evolution from simple analytical expressions to complex computational methods and, most recently, to data-driven models based on artificial neural networks.

The basic classical theory is based on the Euler–Bernoulli hypothesis of plane sections. It represents the foundation of every calculation, including those using neural networks, and enables the determination of the moment–curvature relationship in the elastic phase. This analysis neglects bond-slip between reinforcement and concrete and uses simplified constitutive laws (linear-elastic material response, or bilinear models with limited plasticity) [9]. Despite its simplicity, it defines the basic terminology (neutral axis, curvature, tensile zone, compressive zone) that remains the basis for all subsequent methods.

With the development of modern strength theory and the improvement of concrete and reinforcement models, approaches have incorporated plastic and nonlinear effects. The key challenge in this phase became the selection of an adequate constitutive material behavior law (e.g., Kent-Park, Mander, or Scott-Park models), as subtle differences in the concrete stress-strain curve (especially in the post-peak softening zone) can lead to significantly different values of curvature and ultimate moment capacity. The development of these constitutive models enabled the accurate capturing of concrete cracking, yielding of reinforcement, and ultimate failure of the section, which is crucial for seismic analysis and assessment of existing structures [9].

In engineering calculation practice, numerical methods such as iterative procedures and the finite element method have become the standard for solving the moment–curvature relationship for complex cross-sections and various loading conditions. The discretization phase (Fiber Method) represents the "gold standard" in the analysis of reinforced concrete sections. The section is divided into a large number of fibers – small sub-areas of concrete and reinforcement – where each fiber is assigned a constitutive law, and the section equilibrium is achieved iteratively (most commonly using the Newton-Raphson method). Software implementations such as OpenSees (open-source) and ABAQUS (commercial FEA) enable modeling of practically any cross-section geometry and material [10].

However, classical numerical approaches often face challenges when it comes to generalizing the model to different types of cross-sections or variations in material parameters – especially when dealing with preliminary dimensioning of structures or real-time analyses. Modern numerical developments focus on reducing the "iterative cycle" through model reduction and surrogate models, which directly opens the door to the application of machine learning techniques [1].

The transition to ANN has proven particularly promising due to their ability to model complex nonlinear functions based on datasets obtained from experiments or numerical simulations. ANN is theoretically viewed as a function approximator that "maps" input parameters (geometry and material) directly to output parameters (yield moment, ultimate curvature, ductility) in milliseconds instead of minutes, thereby bypassing the iterative cycle altogether. The practical implication is that ANN does not replace previous numerical methods but rather compresses their results into a rapid interface: the network is trained on data generated by the most precise methods (fiber analysis, FEM) and then enables almost instantaneous prediction for new parameter combinations.

Numerical development has progressed from simple equations, through iterative integration over fibers (OpenSees, ABAQUS), to today's hybrid models where ANN serves as a rapid interface that utilizes knowledge generated by the most precise numerical methods of the past. Modern development of numerical methods clearly demonstrates the tendency of combining the accuracy of the fiber method with the speed of ANN, representing a significant step forward in the real-time application of nonlinear analysis in structural engineering practice.

3. DEVELOPMENT AND APPLICATION OF ANN IN STRUCTURAL ENGINEERING

ANNs are increasingly used in structural engineering as data-driven models for predicting and approximating complex nonlinear relationships in the behavior of concrete structures. These models transform input data into output predictions through layers of interconnected neurons, using mathematical optimization algorithms for learning from the training database [1]. The key characteristic of ANN models is their ability to capture complex nonlinear patterns that are difficult to describe with conventional analytical expressions, making them particularly suitable for predicting mechanical characteristics and load-bearing capacity of concrete elements and structures.

In the early period of application in civil engineering (1989–1994), ANNs primarily served to demonstrate that it was possible to learn nonlinear material behavior rules, i.e., material mechanics, without explicit differential equations. The potential of ANN in civil engineering was originally presented by Adeli (1989), while a comprehensive overview of ANN applications in civil engineering, including numerous references, is provided by Adeli (2001) [11]. One of the earliest applications, Ghaboussi et al. (1991), demonstrated an ANN model for approximating the constitutive behavior of materials under monotonic loading [12]. These papers laid the foundation by showing that ANN can capture the basic nonlinearity of materials without explicitly specifying constitutive equations.

With the development of concrete technology (1995–2005), the focus of ANN application shifted from the general concept to specific predictions of material properties, when they were also used for predicting mechanical characteristics of concrete elements, including the strength and load-bearing capacity of columns and beams. That ANN models are superior compared to regression models in predicting the compressive strength of concrete was demonstrated by Yeh (1998) [13]. Hadi (2003) [14] applied ANN for predicting the load-bearing capacity of concrete columns with and without confinement. A comprehensive overview of ANN application in civil engineering, including numerous references, is provided by Adeli (2001) [11]. During this period, several studies confirmed that ANN can predict the mechanical characteristics of concrete elements with high accuracy ($R^2 > 0.95$).

Later (2005–2015), the focus shifted toward specific structural problems in addition to defining load-bearing capacity, ANNs were also utilized to estimate deformability and ductility, which has been successfully applied in more recent studies to predict failure loads and other performances under extreme loading [15], [16]. Studies have shown that ANNs can recognize complex non-linear characteristics in experimental data without assuming a functional form, making them suitable for modeling the behavior of concrete elements with various input parameters [17]. Research has demonstrated that ANNs can simultaneously process the effects of concrete cover and stirrup spacing, which are parameters that directly affect the confinement level of the concrete core—a factor that is extremely difficult to accurately quantify in traditional fiber models without losing convergence [18]. The trend in the literature also points to the potential application of ANNs in modeling complex relationships such as moment–curvature, where ANNs can serve as an efficient approximation method alongside numerical or experimental approaches [1], [14].

ANNs enable rapid and efficient prediction, which is useful when large datasets from laboratory tests or numerical simulations of concrete behavior are available. These models reduce computation time and improve design parameters, especially when traditional analytical methods become too complex for practical application [17]. During this period, the focus shifted from simply "whether ANN can predict" to "how reliably it generalizes and whether it can replace or complement established engineering models".

In the most recent phase, attention is increasingly directed toward model interpretability, i.e., toward reducing the "black box" effect, which is particularly evident in hybrid ANFIS systems. While early MLP models provided accurate results without explaining the logic, the most recent works (Mangir et al., 2025 [3]) introduce fuzzy rules that enable tracing the decision-making process, thus increasing the transparency and trust of structural engineers in data-driven models.

4. ANALYSIS OF PUBLISHED RESEARCH

In order to examine contemporary approaches to the application of neural networks in moment–curvature relationship analysis, a systematic review of relevant research from available scientific databases was conducted. The review covered relevant databases (Scopus, Web of Science, Google Scholar) with a focus on papers that combine the application of neural networks with the analysis of reinforced concrete cross-sections. Based on the review and analysis of papers, four characteristic studies that represent different methodological approaches were selected and comparatively analyzed.

4.1. Methodological Framework and Review of Relevant Research

Analysis of the four studies in which the process of defining the moment–curvature relationship using ANN is presented reveals that they do not constitute a homogeneous group. Jadid & Fairbairn (1996) [8] deal with an experimental beam-column connection system; Bağcı (2010) [1] and Sakcalı & Yüksel (2022) [7] are predominantly parametric models trained on numerically generated databases; while Mangir et al. (2025) [3] introduce a hybrid ANFIS model that treats curvature as an input variable. In the following text, the results of the four analyzed studies are described.

M. N. Jadid and D. R. Fairbairn (1996) [8] use ANN for solving complex nonlinear mapping of beam-column connections, predicting moment-curvature parameters based on experimental data. The significance of this work is primarily

methodological: it was demonstrated that neural networks can learn the relationship between six geometric and material parameters and the characteristic values of yield curvature (ϕ_y) and yield moment (M_y). The MLP architecture with two hidden layers (6-40-30-2), trained on 272 numerical patterns and 34 experimental samples, achieves an R.M.S. error of only 0.02%, which confirms the extraordinary approximation capability of ANN. Although the model is limited to yield point parameters of beam-column connections (which is a narrow domain), this work was the first to demonstrate the potential of neural networks for predicting moment-curvature parameters and opened the way for all subsequent research in this field.

M. Bağcı (2010) [1] uses ANN for predicting the moment-curvature relationship of reinforced concrete sections with emphasis on nonlinear material behavior. For generating training data for a multi-layer feed-forward network with backpropagation algorithm, an analytical method (filament model) was used. This work belongs to the category of parametric models where the network simultaneously predicts six characteristic parameters of the $M-\phi$ diagram (T_y , T_h , C_v , C_c , C_o , C , ϵ). The accuracy of the model is exceptionally high (average training error < 0.2%), which is expected since the data were generated by a deterministic mathematical model without noise. The contribution of this work lies in the demonstration that ANN can reproduce the complete moment-curvature curve through several key control points, with practically negligible error.

G. B. Sakcalı and İ. Yüksel (2022) [7] analyze reinforced concrete rectangular columns subjected to eccentric compression and biaxial bending. Training data for the network were obtained using software for cross-section analysis based on the fiber method (fiber section analysis). This work is significant because it extends the domain of ANN from simple bending to complex three-dimensional stress states. The model predicts three key parameters (yield curvature ϕ_y , ultimate curvature ϕ_u , and yield moment M_y) with a coefficient of determination R^2 in the range of 0.954 to 0.990. Although the database consists of only 100 samples, the model successfully captures the behavior of columns under biaxial bending.

A. Mangir, V. Okumus, and Z. Şen (2025) [3] introduce a hybrid ANFIS model (Adaptive Neuro-Fuzzy Inference System) for precise prediction of the nonlinear moment-curvature relationship of RC beams. Unlike previous works, this model treats curvature as an input variable, which enables the generation and reconstruction of the entire $M-\phi$ curve. The advantage of the ANFIS model lies in the combination of fuzzy logic and neural networks, which provides better interpretability of results – one of the biggest weaknesses of traditional ANN models. The mean relative error (MRE) is 3.55%, which is higher than in parametric models but justified by the complexity of the task – predicting the entire nonlinear curve instead of just a few characteristic points. This model represents the most modern approach in the analyzed set and demonstrates the tendency toward more transparent and interpretable models.

In order to systematically compare these approaches, and for a clearer understanding of the similarities and differences between the analyzed approaches, their key characteristics are summarized in Table 1, which serves as the basis for further comparative analysis.

Table 1: Comparative overview of ANN models in the analyzed studies

Characteristic	M. N. Jadid and D. R. Fairbairn [8]	G. B. Sakcalı and İ. Yüksel [7]	M. Bağcı [1]	A. Mangir et al. [3]
Structural element	Beam-column connection (biaxial bending)	RC columns (biaxial bending)	RC cross-sections (general model)	RC beams with rectangular cross-section
Network type	MLP Backpropagation	Forward-fed Backpropagation	Multi-layer Feed-forward	ANFIS(Neuro-Fuzzy)
Input variables	6 variables	7 variables	7 variables	6 variables
Architecture	2 hidden layers (6-40-30-2)	1 hidden layer (7-5-3)	2 hidden layers (7-12-13-6)	6 layers (Fuzzy rules)
Activation functions (between layers)	Sigmoid and Hyperbolic tangent	Sigmoid function	Sigmoid function	Gaussian membership functions (Gaussian MF)
Output variables	ϕ_y , M_y (characteristic points)	ϕ_y , ϕ_u , M_y (characteristic points)	6 parameters, characteristic points of T_y , T_h , C_v , C_c , C_o , ϵ , M	Moment (M) for each diagram point, entire MC diagram
Approach type	Parametric / experimental	Parametric	Parametric	Pointwise
Database	272 valid patterns (FORTRAN + 34 experimental samples)	100 samples (MATLAB fiber analysis)	52 data sets (Filament model)	95 cross-sections (OpenSees)
Statistical data	R.M.S. error: 0.02%; Correlation: 1.0	Coefficient of determination (R^2) ranges from 0.954 to 0.990 for three parameters	Average training error: < 0.2%	Mean relative error (MRE): 3.55%

4.2. Databases and Factors of ANN Model Success

Based on the presented models, it can be observed that the success of ANN application in predicting the nonlinear response of cross-sections directly depends on several key factors. First and foremost, the quality and volume of the database represent the foundation for reliable model learning, along with proper selection of input variables and optimization of the training algorithm.

Most researchers use numerically generated databases to overcome the high costs and practical limitations of experimental testing. The dominant method for generating "ground truth" data is fiber (filament) analysis, which discretizes the section into a large number of parallel elements and performs iterative equilibrium through the Newton-Raphson procedure. The accuracy of these numerical data directly determines the upper bound of ANN model accuracy – if the database contains noise (experimental scatter) or systematic bias (simplified material models), the network cannot achieve better performance than the data on which it was trained.

4.3. Model Architecture and Activation Functions

In addition to database quality, the choice of neural network architecture and associated learning algorithms has a decisive influence on the model's ability to approximate complex nonlinear dependencies. In terms of architecture, the most commonly used architecture is the Multi-layer Perceptron (MLP), with one or two hidden layers. MLP with two hidden layers (e.g., 6-40-30-2 in [8] or 7-12-13-6 in [1]) has proven particularly suitable for parametric models where the outputs are characteristic points of the $M-\phi$ curve. In contrast, simpler architectures with one hidden layer and fewer neurons (7-5-3 in [7]) are sufficient when the goal is prediction of a small number of output parameters.

On the other hand, the application of the hybrid Adaptive Neuro-Fuzzy Inference System (ANFIS) model combines the logical reasoning of fuzzy sets with ANN learning. Research shows that ANFIS can achieve better generalization and lower error compared to standard MLP, particularly in modeling the entire nonlinear $M-\phi$ curve. As demonstrated in [3], ANFIS uses Gaussian membership functions and hybrid optimization that combines least squares and gradient descent methods.

The choice of activation function determines the network's ability to model sudden changes in stiffness (e.g., concrete cracking or reinforcement yielding). In the analyzed papers, Sigmoid and Tanh functions were applied as standard for MLP models due to their smooth differentiability. Therefore, the choice of architecture and activation function is not arbitrary – it directly reflects the physical nature of the problem: a smooth but strongly nonlinear relationship with several characteristic points of sudden change (cracking, yielding, crushing).

4.4. Analysis of Input Variables

In addition to model architecture, one of the key aspects of every ANN approach is the selection of input variables that representatively describe the physical behavior of the system. Through comparative analysis of input variables, the main cross-section comprises concrete strength, amount of main reinforcement, and cross-section geometry, which are common to all analyzed models (Figure 1). These variables directly control the internal force equilibrium in the section and represent necessary parameters for any analytical or numerical calculation.

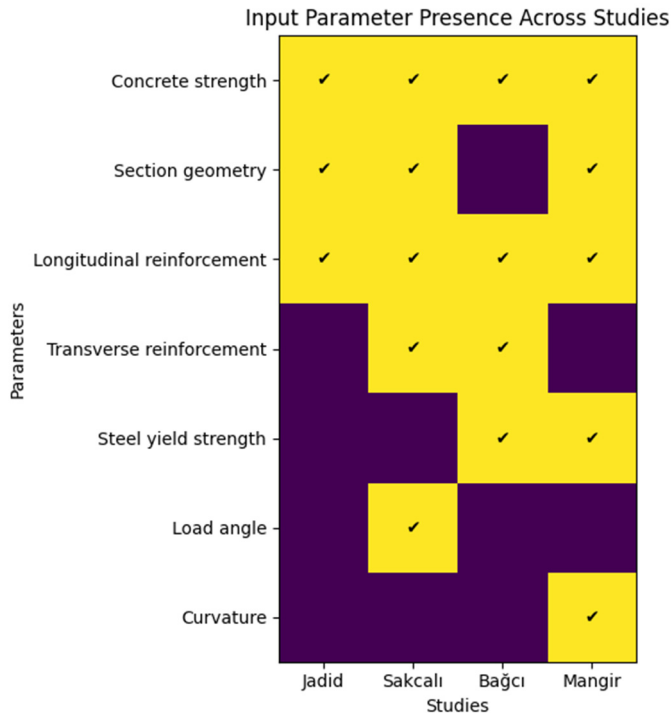


Figure 1: Comparative overview of input variables

Differences arise depending on whether beams are analyzed (where the focus is on the yield strength of steel and the cross-sectional area of reinforcement) or columns (where the influence of normal force dominates). Input parameters vary depending on the complexity of the problem; thus, geometric characteristics, width (b) and effective depth (d), are present in all models, while additional parameters such as concrete cover, confinement, axial force, or curvature value appear depending on the specific problem being analyzed. Models that include a larger number of input parameters generally show greater adaptability but also require larger databases for adequate training.

4.5. Model Typology: Pointwise and Parametric Approach

Based on the previous analysis, it can be concluded that the reviewed works are divided into two methodologically distinct groups. The first group consists of pointwise models, in which curvature is an input variable and the output is the moment for a specific point on the curve. The second group consists of parametric models, in which the network does not reconstruct the curve point by point but rather predicts several key parameters that define the shape of the $M-\phi$ diagram.

Pointwise models predict the moment for a given curvature value and are represented in the work of Mangir et al. (2025) [3], where inputs are cross-section geometry, mechanical material properties, reinforcement area, and the curvature itself, while the output is the moment. The advantage of this approach is that it enables reconstruction of the entire $M-\phi$ curve with arbitrary resolution, but requires a larger training database and can be sensitive to extrapolation outside the range of training data.

In a professional sense, neither of these two approaches can be declared "better" a priori without defining the objective of the analysis. Pointwise models are more suitable when a detailed shape of the entire $M-\phi$ curve is required, whereas parametric models are more suitable when the objective is to rapidly obtain several key indicators for structural analysis, design, or ductility assessment. The most detailed insight into the state of the material within the cross-section, predicting both the crushing points of the concrete cover and core, is presented in the research of M. Bağcı [1]. The parametric approach dominates in the works of Bağcı (2010) [1] and Sakcalı & Yüksel (2022) [7], which focus on deriving key engineering indicators such as stiffness, ductility, and capacity. These models are particularly suitable for seismic design and bilinear idealization due to their compactness, although they do not provide a detailed insight into the complete course of the curve. A similar focus on experimental parameters and the response classification of the beam-column joint is also present in the pioneering work of Jadid & Fairbairn (1996) [8]. Their main advantage is compactness and immediate usability in seismic design and bilinear idealization, while the main limitation is that they do not provide the detailed course of the complete curve. In contrast to them, the research of Mangir et al. (2025) [3] represents a strict pointwise model, aimed at the direct mapping of points and a continuous description of the structural response. The choice between these two approaches depends on engineering needs, i.e., whether the priority is a detailed reconstruction of the response or the efficient determination of key parameters.

4.6. Statistical Indicators and Model Accuracy

In order to evaluate model performance, various statistical indicators were analyzed, including correlation coefficients, R.M.S. error, and mean relative error. Statistical indicators confirm the high reliability of the models and indicate that ANN-based models achieve a level of accuracy that is practically applicable in engineering design. However, direct comparison of accuracy between models requires caution, as each model uses a different database, different input parameters, and different levels of task complexity.

Models that focus on defining characteristic points report exceptionally low errors [1,8]. The highest accuracy is recorded in the research of Bağcı [1]. The reason for this precision lies in the application of purely analytical data (generated via a "filament model") because when an ANN learns mathematically precise functions without noise, it can replicate them almost perfectly. The research of Jadid [8] is a pioneering attempt to demonstrate that an ANN can replace an experiment. Within the analyzed dataset, a very high accuracy was reported, which indicates the ability of the network to correctly classify failure modes and trends based on a limited number of laboratory tests. For columns subjected to biaxial bending in the research of G. B. Sakcalı and İ. Yüksel, the accuracy remains high, but a certain variance among the parameters is observed [7]. It is noted that the ultimate limit state (failure) is more difficult to predict than the yielding state, as the behavior of concrete before failure is highly non-linear and sensitive to small variations in the inputs. It can be said that the ANN is superior when formed on the basis of "clean" data, free of noise, and from fixed mathematical models [1, 7, 8]. It is necessary to emphasize that very high correlation values and very low errors should not be automatically interpreted as a universal superiority of the ANN approach, but always within the context of the database size, the method of splitting into training/test sets, and the fact that many databases are numerically generated, so the ANN primarily approximates the reference numerical model from which it learns.

For models that predict continuous nonlinear dependencies, such as the ANFIS model in the study by A. Mangir et al. [3], the mean relative error (MRE) is approximately 3.55%, while most results fall within the limits of $\pm 5\%$, which suggests that the error is practically acceptable for engineering application, as it is smaller than the typical uncertainty of the constitutive models for concrete and reinforcement materials themselves. However, the interpretation of these indicators must be conducted with caution, taking into account the origin and characteristics of the database on which the model was trained.

4.7. Interpretability, Advantages, and Limitations of Models

In addition to accuracy, contemporary research increasingly emphasizes the importance of interpretability and practical applicability of ANN models in engineering practice. One of the greatest advantages of applying ANNs is the speed of obtaining results. While numerical simulations in software such as ABAQUS are slow and expensive, neural networks provide a non-linear response instantaneously. The practical value of these studies is further highlighted through the development of user tools. Although ANN models demonstrate high accuracy, one of the challenges remains their "black box" nature. Modern papers are beginning to introduce techniques such as Shapley Additive exPlanations (XAI - Explainable AI) to interpret the influence of input parameters, such as cross-sectional depth and reinforcement, on the final capacity prediction. Among the analyzed papers, only one study (Mangir et al., 2025) [3] is observed to have made a significant step toward eliminating the "black box" effect by using fuzzy sets and expert opinions to reduce the rule base, thereby making the decision-making process within the model transparent and verifiable for engineers. It should be emphasized that the speed of ANN / ANFIS models represents a clear practical advantage, but the cost of that speed can be reduced interpretability. Precisely for this reason, the significance of the work by Mangir et al. (2025) [3] is greater than the error percentage itself – that work demonstrates the direction of moving from purely black-box models toward approaches that are more engineering-verifiable and explainable.

Comparative analysis shows a clear tendency: while early models [8] aimed to prove the very possibility of learning, modern approaches [3] are focused on model transparency and speed (results in milliseconds) without loss of physical foundation of the calculation. Statistical indicators of all analyzed works confirm that ANN is no longer just a tool for scientific investigation but a practically applicable tool in everyday engineering practice. An overview of the chronological development of works also reveals an increase in the complexity of the problem being solved – from simple connections [8], through individual sections [1], to complex biaxial bending of columns [7] and complete M- ϕ curves via ANFIS [3].

5. RECOMMENDATIONS FOR FUTURE RESEARCH

Based on the detailed analysis of the papers, it is evident that science has traversed a path from proving that ANN can predict moment-curvature at all [8], through optimization of speed [1], to attempts to make the process logically transparent [3]. Also, based on the analysis, shortcomings as well as possible directions for future research can be identified. In the following text, recommendations for further research are briefly described.

Since all proposed studies primarily deal with the moment-curvature relationship under monotonic loading, it is necessary to investigate the behavior of real structures subjected to cyclic loading. Developing an ANN model that predicts hysteretic loops of the moment-curvature (M- ϕ) relationship, taking into account degradation of stiffness, strength reduction, and energy dissipation through cycles, remains important for application in seismic engineering.

The analyzed models are based on ideal cross-sections, where it is necessary to include parameters such as the degree of corrosion and time as input variables. Reinforcement corrosion drastically changes the moment-curvature (M- ϕ) relationship over time (reduction of steel area, loss of concrete-steel bond, cracking of the protective concrete layer). Therefore, ANN models that predict the moment-curvature relationship should be extended to include time-dependent effects.

A classical cross-section was analyzed, without the possible application of high-strength concrete, concrete with recycled aggregate, or fiber-reinforced polymer (FRP) reinforcement. Analyzing the moment-curvature relationship for hybrid sections (e.g., combination of steel profiles and concrete – SRC) or sections strengthened with FRP strips/sheets would expand the domain of ANN application from conventional RC to modern composite and rehabilitated structures.

In current research, the moment-curvature relationship is treated as an isolated problem, where integration into BIM and real-time simulations is needed. Developing an ANN model that can be integrated as a plugin for BIM software (such as Revit or Tekla) or software for nonlinear analysis (OpenSees, SAP2000) would enable engineers to have an M- ϕ response for any section in real time during the design process. This would transform ANN from a scientific tool into a practical design instrument.

6. CONCLUSION

The application of ANN in defining the nonlinear moment-curvature relationship represents a significant advancement in structural engineering, as it enables bridging the gap between high numerical accuracy and the demands for rapid engineering application. Through analysis of model evolution – from pioneering MLP networks to modern ANFIS systems – it can be concluded that the development direction is clearly defined: toward faster, more accurate, and more transparent predictive models.

Key findings of the paper indicate that:

- Computation speed in milliseconds enables optimization of thousands of cross-sections in real time, which is computationally too expensive with classical numerical methods (such as the Newton-Raphson procedure);
- ANN models have proven superior to linear regression because they naturally model the pronounced nonlinearity of concrete and steel, particularly in the yielding and ultimate failure zones;

- Database quality remains the primary factor of accuracy, where numerically generated data (e.g., via OpenSees) are used as "ground truth" for training robust predictive models.

Viewed as a whole, the analyzed works demonstrate that development in this field progresses from proving the very possibility of ANN approximation, through optimization of architecture and database, to modern models with a greater degree of interpretability. Thus, ANN and ANFIS are not imposed merely as faster tools but as a bridge between the precision of classical numerical methods and the requirements of modern engineering practice for speed and transparency.

Future research directions must focus on eliminating the "black box" effect through XAI (Explainable AI) techniques, integration into BIM workflows, and introduction of time and corrosion parameters into input variables. This will enable neural networks to cease being merely an alternative tool and become a standard part of the engineering design process, thereby contributing to safer, more efficient, and more economical concrete structures.

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